B.Sc Part I (Physics Hons)
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Question :- (a) State and Explain the (basic) fundamental postulates of the special theory of relativity.

(b) A reference frame S moves w.r.to another frame S' with uniform velocity $\vec{v}$. Derive Lorentz space and Time Transformation eqn gives x', y', z', t' in terms of x, y, z, t the moving frame coincides with stationary ones at $t' = t = 0$. Prove that when $\vec{v}$ is much smaller than velocity of light Lorentz Transformation reduce to Galelian Transformations.

Ans :- (a) **Special theory of relativity :-**

The special theory of relativity was enunciated in 1905 by Albert-Einstein. It has two fundamental postulates :-

(1) The laws of physics are invariant in all inertial systems :- An inertial system is defined as a co-ordinate frame of reference within which the law of inertia, i.e Newton's first law of motion holds. A body on which no net external force acts will move with a uniform velocity, if it is in an inertial system. Hence according to this postulates the mathematical form of physical law remains the same for any two observers moving with constant linear velocity relative to eachother. It is therefore not possible to distinguish one inertial system from another by an experiment in physics, As the laws of physics are the same for an inertial frame. In other words there is no preffered inertial system.

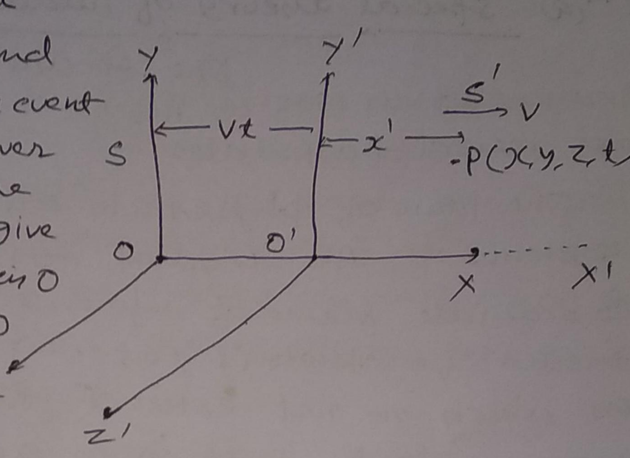
(11) The speed of light in vacuum is a constant independent of the inertial system, the source and the observer. In other words, the velocity of light is an invariant.

(b) **Lorentz Transformation :-**

The eqn in relativity physics, which relate the space and time co-ordinates of two co-ordinate systems moving with a uniform velocity relative to one another are called Lorentz transformation.

Consider two observer O and O' located in two seperate inertial Co-ordinate system S and S'. The system S' moves with a uniform velocity v to the right angle along the X-axis relative to S. This is equivalent to the motion of S to the left with a velocity v relative to S'. Suppose each observer carries a metre rod and a clock to measures the position and time of a particle relative to an inertial system. By specifying the position and time of a physical phenomenon, the observer describes what is called an event

The space and time Co-ordiniates of an event at P described by an observer O are (x, y, z, t) and the co-ordeniates of the x, y, z give the distance from the origin O along the X, Y & Z directions as measured by the meter stick of the observer O and t gives the time



that he reads on his clock. Suppose both the observers are temporarily at rest w.r.to each other when they compair their metre sticks and sychronise their clocks. The system S' is then set in motion w.r.to system S. when the origin of S' passes the origin of S both the clocks reads zero, i.e $t=0$ and $t'=0$ and at that instant $x=x'$

It is evident that after time t as measured by O, the origin of the system S' is at distance vt from the origin of the space S.

$$\therefore \; x' = x - vt$$

As the relative motion between S' and S is at right angle to Y and Z axes, the position Co-ordeniate

$$y' = y \text{ and } z' = z$$

Further according to classical physics

the two observers O and O' will compute the same time for any signal orginating from P after giving allowance for the velocity of S'

$\therefore t' = t$

Hence non relativistic Galelian Transformation eq^n are.

$x' = x - vt$, $y' = y$, $z' = z$, $t' = t$

Lorentz Transformation eq^n which satisfy the relativity requirements according to the two postulates must also be linear and not quadratic or of a higher order. A quadratic equation has two roots and a higher order equation even more. In order that an event (x, y, z, t) in the inertial system S may correspond to a single event (x', y', z', t') in the inertial system S' and vice-versa, there must be a one to one correspondance and the transformation equations must be linear in space co-ordinates as well as in time coordinates.

As the inertial system S' is moving with respect to the inertial system S along the X-direction, A ray of light parallel to the X-axis remains always parallel to X'-axis

$Y = y'$ and $Z = Z'$

Hence x and t will both depend on x' and t'.

We shall therefore assume the simplest linear equation for Lorentz transformation, i.e

$x = Ax' + Bt'$, $y = y'$, $z = z'$ and $t = Gx' + Ht'$.

It has been assumed in the above eq^n that it is possible that the time interval for the two observer O and O' in the inertial system S and S' may not be identical

Imagine that at time $t = t' = 0$, when the origin O' coincides with the origin O a spherical pulse of light leaves the common origin of S and S'. As the velocity of light is invariant each observer sees a special wave expanding outwards with the speed c in his own system as measured by his own meter stick and clock.

$\therefore$ For the observer O in system S $c = \sqrt{x^2+y^2+z^2}/t$

or. $x^2+y^2+z^2 = c^2t^2 \Rightarrow x^2+y^2+z^2-c^2t^2 = 0$ ------ (1)

Similarly for observer O' in system S', $c = \sqrt{x'^2+y'^2+z'^2}/t'$ ---- (II)

$\therefore x'^2+y'^2+z'^2-c^2t'^2=0$

from (I) and (II) $x^2+y^2+z^2-c^2t^2 = x'^2+y'^2+z'^2-c^2t'^2$, $y=y'$ & $z=z'$

$\therefore x^2-c^2t^2 = x'^2-c^2t'^2$

Substituting $x = Ax'+Bt'$ and $t = Gx'+Ht'$ we get

$(Ax'+Bt')^2 - c^2(Gx'+Ht')^2 = x'^2-c^2t'^2$

comparing co-efficients on both sides we have ---- (III)

$x'^2[A^2-c^2G^2] = x'^2$ or $A^2-c^2G^2=1$ ---- (IV)

$t'^2[B^2-c^2H^2] = -c^2t'^2$ or $B^2-c^2H^2=-c^2$ ---- (V)

& $2x't'[AB-c^2GH]=0$ or $AB-c^2GH=0$

Now when $x'=0$, $x=vt$

$\therefore x = Ax'+Bt'$ and $t = Gx'+Ht'$, we get $vt=Bt'$ & $t=Ht'$

$\therefore vHt' = Bt'$ $\therefore B = vH$

from (IV) $v^2H^2-c^2H^2=-c^2$ or $H^2 = \frac{c^2}{c^2-v^2}$ $\therefore H = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Substituting this value of H in $B = vH$, we $B = v/\sqrt{1-\frac{v^2}{c^2}}$

from (V) $B=vH$ (we get)

$AvH - c^2GH = 0$ or $Av = c^2G$ $\therefore G = \frac{Av}{c^2}$

Substituting $G = Av/c^2$ in eq^n (III)

$A^2 - A^2\frac{v^2}{c^2} = 1$ $\therefore A = 1/\sqrt{1-\frac{v^2}{c^2}}$

Hence $G = \frac{v/c^2}{\sqrt{1-v^2/c^2}}$

Substituting these values of A, B, G & H, the Lorentz transformation are

$$x = \frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{t'+\frac{v}{c^2}x'}{\sqrt{1-v^2/c^2}}$$

$y = y'$ and

$z = z'$